Transient-State Modeling of Distribution Transformers

Mehdi Bigdeli¹, Ebrahim Rahimpour², Masoud Khatibi³

Abstract – In this paper, a black box model is proposed to analyze the transient state of distribution transformers. This model is capable of showing the frequency characteristics of the transformer under various conditions of the terminals connections up to the frequency domain of almost 1.2 MHz. In addition, this model has satisfactory abilities in illustrating the characteristic of the wave transferred to the consumer side and it can be employed in studying the over voltages produced in the transformers terminals as the result of striking the lightning waves. In order to investigate the validity as well as the accuracy of the proposed model, a 6300/420 V, 2500 KVA distribution transformer is selected for the research. First, required experiments are performed on this transformer and then, using the measurements results and applying the modal analysis, the model parameters are obtained. Comparing the results of the model and the measurements findings shows that the proposed model can be utilized for transient studies in distribution networks. Copyright © 2011 Praise Worthy Prize S.r.l. - All rights reserve.

Keywords: Distribution Transformer, Transient State, Black Box model, Modal Analysis

Nomenclature
C: Capacitance
L: Inductance
R: Resistance
FFT: Fast Fourier Transform
Y₁: Primary winding admittance
Y₂: Secondary winding admittance
Yₚₛ: Mutual admittance between primary and secondary winding
Yₚ₉: Primary winding admittance to ground
Yₛ₂: Secondary winding admittance to ground
Yₘₙₙ: Magnetizing admittance
Zₖₛₚ: Primary skin effects impedance
Zₖₛₛ: Secondary skin effects impedance
n: Turn ratio of ideal transformer
fᵦ: Resonance frequency
Q: Quality factor

I. Introduction

Over voltages, which are produced in power systems because of various factors such as lightning, switching, and other disturbances like short circuits or connecting the no-load transformer through a cable, have a frequency spectrum from zero to several mega hertz that in the case of correspondence between one of the existing frequencies in this voltage with one of the natural frequencies of the transformer, the phenomenon of resonance will occur in the transformer windings. This phenomenon can cause the winding insulation to be locally under the electrical tension which may damage it. Besides, over voltages in the transformer terminals can increase the transient voltages in different parts of a power system.

Hence, the existing methods for the transient-state analysis of transformers can be categorized based on two viewpoints: first, from the standpoint of transformer design engineers—interested in physical models—and second, from the outlook of power system design engineers—interested in black box models. In the former one, the oscillating behavior and the electrical tensions in the windings are focused. In the latter, the current and voltage waveforms in transformer terminals are in the focus and it is employed whenever the over voltages and the transient-states are studied in the power system.

Distribution transformers are of the most important transformers which are frequently utilized in distribution and sub-transmission networks. For their suitable performance during the operation, these transformers should be resistant against various tensions such as the tensions resulted from the impulse and the lightning waves. Thus, studying the transient-state of these transformers is of great importance. Although a number of studies have been performed so far in the context of the transformer transient-state modeling [1]-[13], distribution transformers have not been paid sufficient attention in this sort of research studies [14].

Generally, the existing models related to the transient-state analysis of transformers can be categorized in three groups:

- Black box models:
  - Modal analysis [1], [2].
  - System description by means of zero and pole position [3].

- Physical models:
  - Detailed model [4]-[6].
• Multi-phase transmission line model [7]-[10].

In order to perform the various analyses of distribution systems, researchers interested in these types of systems need a black box model of transformers which includes the following evident characteristics:

1) Adequate accuracy,
2) Relative simplicity from the computational and hence less simulation time viewpoints.

Therefore, a black box model is introduced in this paper which satisfies the above-mentioned conditions. Since in the black box modeling as the system description by means of zero and pole positions the result of the system identification process is a transfer function and not an equivalent circuit, and as a circuit model is more understandable in most of the cases, the modal analysis is utilized and focused in this paper. In this method, the eigenfrequencies of the system's transfer function describes its oscillating behavior. A series resonant circuit can be related to an eigenfrequency. Thus, the modal analysis will result in the description of an oscillating system as the parallel connection of series resonant circuits. The number of series resonant circuits depends on the number of the system eigenfrequencies. The values of RLC elements are determined from the measurement results.

In order to investigate the validity as well as the accuracy of the proposed model, a 2500 KVA, 6300/420V distribution transformer is selected for the research. First, required experiments are performed on this transformer and then, using the measurements results and applying the modal analysis, the model parameters are obtained. Comparing the results of the model with those of the measurements demonstrates that the proposed model can be employed in various research studies such as the study of over voltages produced as a result of striking the lightning waves to the terminals of the utilized transformer.

II. The Proposed Model

Fig. 1 illustrates the model proposed in this paper. The model includes the following components:

1) An ideal transformer to consider the voltage ratio,
2) \( Y_{\text{mag}} \), to consider the no-load current including the magnetizing current and the currents related to the core losses.
3) \( Z_{\text{skin1}} \) and \( Z_{\text{skin2}} \), the primary and the secondary impedances, respectively, including conductors skin effects.
4) \( Y_1 \) and \( Y_2 \), to consider self inductance and also series capacitance of primary and secondary winding.
5) \( Y_{\text{mut}} \), to consider the mutual inductance and the capacitance between the two windings, which is mainly capacitive.

6) \( Y_{s1} \) and \( Y_{c2} \), to model the capacitance of the individual windings to the ground, which has also inductive effect in high frequencies.

As it is shown in Fig. 1, the number of model elements cannot be determined from the beginning and they can be specified once the experiments are performed and the frequency characteristics of the admittances are plotted. This is because of the fact that in the modal analysis, an RLC series circuit is considered per a resonance in the measured admittance. Estimating all of parameters for the above equivalent circuit to complete the modeling can be done by means of the various experiments results, which will be explained thoroughly in the following sections.

III. Circuits and Measurements Results

III.1. The measurement technique

The measurements required for the specification of the transfer function can be performed both in the time and the frequency domains. The executed research studies confirm that the accuracy of the both methods is equal [15]. In the research executed in this paper, the measurements to specify the transfer function are performed in the time domain. In this method, the impulse voltage is applied to one of the transformer terminals and the values of voltages and currents of the other terminals and even the current of the terminal under the impulse voltage are measured as the outputs and the applied impulse voltage is measured as the input. Fig. 2 schematically depicts the fundamental of measuring the transfer function in the time domain.

All of the considered experiments have been performed in the high-voltage laboratory of Iran Transfo Company in Iran. In these experiments, time domain signals are excited and the related response is sampled after filtering. Next, the signals spectrum is specified via the Fast Fourier Transform (FFT) and finally, the complex transfer function is resulted as the amplitude and the phase by dividing the output signal spectrum to the input signal spectrum.
III.2. Measuring the $Y_{sm}$, $Y_{s1}$, and $Y_{s2}$ capacitive admittances

The values of $Y_{sm}$, $Y_{s1}$, and $Y_{s2}$ admittances cannot be measured directly but they can be calculated by measuring the admittances called $Y_{c1}$, $Y_{c2}$, and $Y_{c3}$ via three appropriate measurement circuits. Fig. 3 illustrates the circuits required to measure the $Y_{c1}$, $Y_{c2}$, and $Y_{c3}$ admittances.

Fig. 4 depicts the measured frequency characteristic of the $Y_{c1}$ admittance. It can be observed that in the considered characteristic two resonances are observed which each resonance can be modeled with a series RLC circuit using the modal analysis. The calculation of $Y_{sm}$, $Y_{s1}$, and $Y_{s2}$ admittances through the measured $Y_{c1}$, $Y_{c2}$, and $Y_{c3}$ admittances are explained in the following section.

Fig. 3. Various circuits to measure the capacitive admittances
It should be noted that based on Fig. 7, the measured admittances in Fig. 6, i.e. \( Y_{\text{short1}} \) and \( Y_{\text{short2}} \), also include the \( Y_{\text{sm}} \), \( Y_{\text{s1}} \), and \( Y_{\text{s2}} \) in the high-frequency part. Thus, the \( Y_1 \) and \( Y_2 \) admittances are obtained by subtracting these admittances from the measured admittances.

The results of \( Y_{\text{short1}} \) and \( Y_{\text{short2}} \) measurements are presented in Fig. 8. These results represent the existence of three considerable resonances in these admittances.

III.4. Investigating the effects of U and V phases on each other

All of the above-mentioned measurements are performed on the U and V phases (a side phase and a center phase) of the transformer. These measurements reveal that despite a difference in the position of the side and the center arms, the measured transfer function in these phases are almost the same. One of the measured admittances in the U and V phases are compared with each other in Fig. 9 as a sample.

Hence, it can be concluded that performing the required experiment on one phase is adequate to make the model.

IV. Estimation of the Model Parameters

In this section obtaining the model parameters using the measurements results is investigated.

IV.1. Calculation of the admittances \( Y_{\text{sm}} \), \( Y_{\text{s1}} \), and \( Y_{\text{s2}} \)

The phase characteristics of the \( Y_{c1} \), \( Y_{c2} \), and \( Y_{c3} \) transfer functions (which include two distinct resonances) demonstrate that these functions, before the first resonance and also between the two resonance frequencies, are pure capacitances, which here are called \( C_1 \), \( C_2 \), and \( C_3 \). Because of the existence of two resonances, two RLC branches and hence, two capacitor values should be calculated. For the first resonance, the capacitance values are calculated in the frequency of 100 KHz (before the first resonance) and for the second resonance, these values are calculated in the frequency of 500 KHz (between the two resonance frequencies). Once the \( C_1 \), \( C_2 \), and \( C_3 \) are calculated, the values of \( C_{\text{sm}} \), \( C_{\text{s1}} \), and \( C_{\text{s2}} \) are obtained using (1). (Proving these relations is presented in the Appendix.)

\[
C_{\text{sm}} = \frac{1}{2} \left( C_1 - \sqrt{(C_1 - C_2)(C_2 - C_3)} \right) \quad (1-a)
\]
\[
C_s = \frac{C_1}{2} - C_{\text{sm}} \quad (1-b)
\]
\[
C_{s2} = \frac{C_2}{2} - C_{\text{sm}} \quad (1-c)
\]
To calculate the resistance and the inductance of $Y_{sm}$, $Y_{1}$, and $Y_{2}$ admittances, another parameter called $k_{ij}$ is defined as follows [2]:

$$k_{ij} = \frac{A_{ij}}{2\pi f_{i,j} C_{ij}}$$ \hspace{1cm} (2)

$$f_{ave,j} = \frac{\sum_{i=1}^{3} f_{i,j}}{3}, \quad k_{ave,j} = \frac{\sum_{i=1}^{3} k_{i,j}}{3}$$ \hspace{1cm} (3)

In the above relations, the first index ($i=1, 2, 3$) are related to the index of $C_{1}$, $C_{2}$, and $C_{3}$ capacities and the second index ($j=1, 2$) determines the number of resonances. Moreover, $f$ is the value of resonance frequency and $A$ is the admittance magnitude in the resonance frequency. In addition, according to what previously mentioned, $C_{ij}$ is the value of $C_{i}$ capacity in the $j$th resonance frequency.

Assumption that the resonance is occurred in the frequency of $f_{ave,j}$, the values of other parameters of the admittances can be calculated as follows:

$$R_{sp,j} = \frac{1}{2\pi k_{ave,j} f_{ave,j} C_{sp,j}}, \quad p = 1, 2, m$$ \hspace{1cm} (4)

Clearly, $C_{sp,j}$, $L_{sp,j}$, and $R_{sp,j}$ are the parameters of $Y_{sm}$, $Y_{1}$, $Y_{2}$ admittances in the $j$th resonance per different values of $p$. The final calculation results of these parameters are presented in Table I.

<table>
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<tr>
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<th>Second resonance</th>
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<td>27.6</td>
<td>24.9</td>
</tr>
<tr>
<td>$C_{2}$</td>
<td>37</td>
<td>138.1</td>
</tr>
<tr>
<td>$C_{m}$</td>
<td>208.3</td>
<td>51.3</td>
</tr>
<tr>
<td>$L_{1}$</td>
<td>12.9</td>
<td>0.78</td>
</tr>
<tr>
<td>$L_{2}$</td>
<td>9.6</td>
<td>0.35</td>
</tr>
<tr>
<td>$L_{m}$</td>
<td>1.7</td>
<td>0.79</td>
</tr>
<tr>
<td>$R_{1}$</td>
<td>4176</td>
<td>1789</td>
</tr>
<tr>
<td>$R_{2}$</td>
<td>2594</td>
<td>309</td>
</tr>
<tr>
<td>$R_{m}$</td>
<td>535</td>
<td>511</td>
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<thead>
<tr>
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<th>Second resonance</th>
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<tr>
<td>$C_{2}$</td>
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<td>$C_{m}$</td>
<td>208.3</td>
<td>51.3</td>
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<tr>
<td>$L_{1}$</td>
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<td>$R_{2}$</td>
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<td>309</td>
</tr>
<tr>
<td>$R_{m}$</td>
<td>535</td>
<td>511</td>
</tr>
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IV.2. Calculating the parameters of the skin effect impedance

In order to model the skin effects, i.e. the value determination of the parameters in the circuit of Fig. 5 via the measurement results of Fig. 8, it is required the number of parallel RL branches is specified in the first step. In [16] it is mentioned that at least four RL parallel circuits should be utilized to model the skin effect impedance. Therefore, and to enhance the accuracy of the model in high frequencies, four RL parallel circuits are employed to model the skin effect impedance of the primary and the secondary. $R_{0}$ is the d.c. resistance of the winding. In order to obtain $R_{1}, \ldots, R_{4}$ and $L_{1}, \ldots, L_{m}$, the value of the measured impedance $Z_{m} = R_{m} + jX_{m}$ is put equal with the equivalent impedance of Fig.5 in $f = f_{c}$. In this case, we will have an optimization problem which the Genetic Algorithm (GA) is employed to solve it. To run the GA program, the Genetic algorithm toolbox in MATLAB is utilized [17]. The results of running the GA program are presented in Table II.

<table>
<thead>
<tr>
<th>Parameters</th>
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<tbody>
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<td>$R_{1}$</td>
<td>769.5</td>
<td>196.9</td>
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<tr>
<td>$L_{1}$</td>
<td>0.366</td>
<td>7.6</td>
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<td>$R_{2}$</td>
<td>1109</td>
<td>239.3</td>
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<tr>
<td>$L_{2}$</td>
<td>0.605</td>
<td>9.5</td>
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<tr>
<td>$R_{3}$</td>
<td>403.8</td>
<td>264.2</td>
</tr>
<tr>
<td>$L_{3}$</td>
<td>3.3</td>
<td>8.7</td>
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<tr>
<td>$R_{4}$</td>
<td>542.8</td>
<td>270.9</td>
</tr>
<tr>
<td>$L_{4}$</td>
<td>3.1</td>
<td>10.3</td>
</tr>
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IV.3. Calculating the parameters of $Y_{1}$ and $Y_{2}$ admittances

To obtain the parameters $Y_{1}$ and $Y_{2}$, the modal analysis is employed. In this method, the low-amplitude resonances are neglected and hence, the number of unknown parameters is reduced. In the modal analysis, the responses of a circuit to various resonances are analyzed independently from each other and an RLC circuit is considered for each resonance. If the measured frequency characteristic includes n resonances, the parameters of the regarded admittances can be determined using the following relations [1]:

$$R = \frac{1}{f_{max}}, \quad L_{i} = \frac{R_{Q_{i}}}{\omega_{i}}, \quad C_{i} = \frac{1}{\omega_{i} R_{Q_{i}}}$$ \hspace{1cm} (5)

$$\omega_{i} = 2\pi f_{w_{i}}, \quad Q_{i} = f_{w_{i}} / \left| f_{c} - f_{w_{i}} \right|$$

Where $f_{w_{i}}$ is the resonance frequency, $Q_{i}$ is the quality factor, and $f_{w_{i}}$ as well as $f_{c}$ are the 3db cutoff frequencies related to the $i$th resonance where $i=1, 2, 3$ represents the resonance number. Calculation results are presented in Table III.

<table>
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<th>Second resonance</th>
<th>Third resonance</th>
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<td>2314</td>
<td>4021</td>
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<tr>
<td>$L$</td>
<td>0.635</td>
<td>4.5</td>
<td>0.49</td>
</tr>
<tr>
<td>$C$</td>
<td>428</td>
<td>16.8</td>
<td>75.8</td>
</tr>
<tr>
<td>$R$</td>
<td>289</td>
<td>452</td>
<td>515</td>
</tr>
<tr>
<td>$L$</td>
<td>0.98</td>
<td>1.23</td>
<td>0.50</td>
</tr>
<tr>
<td>$C$</td>
<td>41.4</td>
<td>2.9</td>
<td>5.1</td>
</tr>
</tbody>
</table>
IV.4. Calculating the Parameters of no-load Branch $Y_{mag}$

The parameters of $Y_{mag}$ are directly computed from the results of no-load test results. These parameters can be determined from below relations:

$$R_c = \frac{V_n^2}{P_{nl}}$$
$$L_m = \frac{V_n^2}{2\pi f I_n}$$

Where:

$$I_n = \left( I_{nl}^2 - \left( \frac{V_n}{R_c} \right)^2 \right)$$

$V_n$: rated voltage, $f$: power frequency, $P_{nl}$: no-load power and $I_{nl}$: no-load current.

According to above, these parameters are computed as $R_c = 10.7\,\text{k}\Omega$ and $L_m = 3.53\,\text{H}$.

V. Investigation of the Model Validity

Later than the parameters estimation, the measured and the calculated results are compared with each other to investigate the amount of the accuracy and validity of the model. In this investigation, two types of comparisons are made. In the first type, the precision of modeling the admittances or the impedances, as a part of the whole model, are evaluated while in the second type of comparison, the accuracy of the total model is taken into consideration.

In Fig.10, the amount of calculated $Y_{c1}$ and $Y_{short2}$ adaptation with the corresponding measured values is illustrated as a sample.

It is clear that despite a satisfactory adaptation in the frequency characteristics of the model components (Fig.10), the correct responding of the total model to various possible applied excitations is also necessary. Therefore, applying an impulse voltage (which models the lightning wave) to the primary side of the transformer and calculating the voltage transferred to its secondary side using the EMTP software, the precision of the model is investigated. Fig.11 depicts the experimented circuit. The following relation is used to model the lightning wave in the EMTP-ATP [18]:

$$v(t) = v_m \left(e^{-At} - e^{-Bt}\right)$$

$A = 14700, \; B = 2470000$

Where $v_m$ is the amplitude of the impulse voltage and equals to 7.67 kV.

The voltage transferred to the secondary side is calculated by the model and in Fig. 12; it is compared with the measured voltage. Fig. 12 as well as other comparisons represents the acceptable validity of the proposed model. In addition, it shows that the proposed model can be utilized with a high accuracy in studies related to the lightning over voltages.
VI. Conclusion

Regarding the need of distribution networks engineers to a simple as well as an accurate model of distribution transformers to analyze the transient states, a suitable black box model for this kind of transformers is proposed in this paper. The introduced model is able to show the frequency characteristics of the transformer under various conditions of the terminals connections up to the frequency domain of almost 1.2 MHz. In addition, the considered model can be employed in studying the over voltages produced in the transformers terminals as the result of striking the lightning waves. The validity as well as the accuracy of the model is investigated by means of performing appropriate experiments on a 2500 KVA, 6300/420V distribution transformer. Comparing the results of the model with those of the measurements reveals the accuracy and the precision of the proposed model.

Appendix

To prove (1), we act as follows:

If the circuit presented in Fig. 3(a) is applied to the proposed model, the following equivalent circuit will be resulted:

\[ C_1 \rightarrow 2C_{s1} \rightarrow 2C_{sm} \]

Based on the above figure, (1-b) is resulted as follows:

\[ C_1 = 2C_{s1} + 2C_{sm} \Rightarrow C_{s1} = \frac{C_1}{2} - C_{sm} \]

If the circuit presented in Fig. 3(b) is applied to the proposed model, the following equivalent circuit will be resulted:

\[ 2C_{sm} \rightarrow C_{s1} \rightarrow 2C_{s2} \rightarrow C_2 \]

Using this figure, (1-c) can also be resulted:

\[ C_2 = 2C_{s2} + 2C_{sm} \Rightarrow C_{s2} = \frac{C_2}{2} - C_{sm} \]

Similarly, using Fig. 3(c) and applying it to the introduced model, the following equivalent circuit can be obtained:

\[ C_3 \rightarrow 2C_{sm} \rightarrow 2C_{s1} \rightarrow 2C_{s2} \]

Hence:

\[ C_3 = 2C_{s1}C_{s2} + 2C_{sm} \]

Substituting the values of \( C_{s1} \) and \( C_{s2} \) respectively from (1-b) and (1-c), we will have:

\[ C_1 = \frac{(C_1 - 2C_{sm})(C_2 - 2C_{sm})}{C_1 + C_2 - 4C_{sm}} + 2C_{sm} \]

Arranging the above relation, a quadratic equation based on \( C_{sm} \) is obtained as follows:

\[ 4C_{sm}^2 - 4C_{sm}C_1 + C_1C_2 + C_1C_2 = 0 \]

With solving the above equation, the value of \( C_{sm} \) is obtained. Clearly, the presented quadratic equation will have two answers, but the larger value is not acceptable, since it causes the values of \( C_{s1} \) and \( C_{s2} \) to be negative. The smaller value is the same value given in (1-a).

Acknowledgements

This work was supported by Iran Transfo Company in Zanjan, Iran.

References

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Ebrahim Rahimpour was born in 1971 in Bijar, Iran. He got his B.Sc. in electrical engineering from Tabriz University in 1993 and M.Sc. and Ph.D. in electrical power engineering from faculty of engineering of Tehran University in 1995 and 2002 respectively. He received a German Academic Exchange Service (DAAD) scholarship in 1998 and worked at the Institute of power transmission and high voltage technology of Stuttgart University of Germany from 1999 to 2001. He was an associate professor at Zanjan University, working on modeling and monitoring of electrical machines, especially transformers from 2002 to 2007. He received a Georg Forster Research Fellowship of the Alexander von Humboldt Foundation (AvH) scholarship in 2007 and performed some researches about Transfer Function Method at the Institute of power transmission and high voltage technology of Stuttgart University until 2008. Currently, he is working on transformer transients at ABB AG, Power Products Division, Transformers, Bad Honnef, Germany. Dr. Rahimpour is a senior member of the IEEE.

Masoud Khatibi received the B.Sc. and M.Sc. degrees in electrical engineering from Islamic Azad University, Iran, in 2006 and 2009, respectively. He is currently with the electrical engineering department of Islamic Azad University. His current research interests include the study of power systems interfaced with distributed generation technologies as well as the transient stability studies of power systems at the presence of distributed generation.